Available online at www.sciencedirect.com

SCIENCE DIRECT®

International Journal of Heat and Mass Transfer 47 (2004) 5511–5522

International Journal of HEAT and MASS TRANSFER

www.elsevier.com/locate/ijhmt

Absorption of thermal radiation in large semi-transparent particles at arbitrary illumination of the polydisperse system

Leonid A. Dombrovsky *

Institute for High Temperatures of the Russian Academy of Sciences, NCHMT, Krasnokazarmennaya 17A, Moscow 111116, Russia

Received 26 January 2004; received in revised form 28 April 2004
Available online 25 September 2004

Abstract

An approximate theoretical model for nonuniform absorption of the external thermal radiation in a large semi-transparent spherical particle is suggested. As applied to heat transfer problems with diffuse radiation in the wide spectral range, the asymmetric illumination of single particle is considered at each spectral interval as a uniform illumination from backward and forward hemispheres (with respect to the direction of spectral radiation flux). The Mie theory is employed in calculations for particles illuminated from a hemisphere. The modified differential approximation suggested earlier by the author is used in the case of spherically symmetric illumination. Approximate analytical relations for distribution of absorbed radiation power inside a particle are obtained. Results of calculations for typical polydisperse sprays of water and diesel fuel droplets are presented.
© 2004 Elsevier Ltd. All rights reserved.

Keywords: Thermal radiation; Infrared; Absorption; Particle; Droplet; Spray

1. Introduction

The disperse systems of separate spherical particles or droplets placed randomly in vacuum or gas are considered. It is assumed that average distance between the particles is large in comparison with their size and the wavelength of external thermal radiation. In this case, one can employ the radiation transfer theory to calculate radiation field in the medium and absorption of the radiation by particles. In such calculations, the particles are usually assumed isothermal and the volume distribution of absorbed radiation power inside particles is not considered [1–3].

An analysis of nonuniform radiation field inside semi-transparent particles is important not only in the case of considerable thermal radiation from nonisothermal particles [4,5] but also for comparatively cold particles when absorbed radiation affects chemical or phase conversions in the particle. One should mention the problem of heating and evaporation of fuel droplets in diesel engines [6–10] and similar problem for water droplets in spray curtains used in fire shielding [11–15]. The thermal radiation from water or fuel droplets is negligible and the solution is divided in two stages: the ordinary spectral calculation of radiation transfer in disperse system and calculation of volume distribution of absorbed radiation power in semi-transparent droplets.

The monochromatic radiation field inside spherical particle can be calculated by consideration of the incident radiation as a combination of plane electromagnetic waves of different amplitudes from different directions.

* Corresponding author. Tel.: +7 095 362 5590/250 3264; fax: +7 095 362 5590.
E-mail address: dombr@online.ru

0017-9310/$ - see front matter © 2004 Elsevier Ltd. All rights reserved.
doi:10.1016/j.ijheatmasstransfer.2004.07.001

Nomenclature

a	particle radius
A	coefficient defined by Eq. (26)
B_λ	Planck's function
B, B_1, B_2	coefficients in Eq. (33)
C	coefficient in Eqs. (33) and (39)
c_k, d_k	Mie coefficients
D	dimensionless radiation diffusion coefficient
E_r, E_θ, E_φ	amplitudes of electric field components
E_0	electric field amplitude in the incident wave
e_r, e_θ, e_φ	dimensionless amplitudes of electric field components
f	function (32) or (33)
F	particle size distribution
I_0, g_0	functions proportional to the spectral radiation energy density
I	radiation intensity
m	complex index of refraction
n	index of refraction
p, P	absorbed radiation power, power absorption function
$\bar{p}$	dimensionless power absorption function
q	radiation flux
Q_a	absorption efficiency factor
$Q_\mathrm{s}^\mathrm{tr}, Q_\mathrm{tr}$	transport efficiency factors of scattering and extinction
r	radial coordinate inside the particle
$\bar{r}$	dimensionless radial coordinate
$\vec{r}$	vector coordinate in the disperse system
s, S	dimensionless absorption functions defined by Eqs. (10) and (11)
T	temperature
w, W	normalized distribution of the absorbed power defined by Eqs. (17), (18) and (41)
x	diffraction parameter
y	independent variable in Eqs. (7) and (34)
z	independent variable in Eq. (7)

Greek symbols

α	relative value of absorption defined by Eq. (36)
β	dimensionless parameter in Eq. (40)
γ, ν, ζ	parameters defined by Eq. (26)
θ	angle measured from the incident wave direction
κ	index of absorption
λ	radiation wavelength
μ	angular coordinate, $\mu = \cos\theta$
μ_*	value of μ defined by Eq. (21)
π_k, τ_k	Mie angular functions
$\Sigma_\mathrm{a}, \Sigma_\mathrm{tr}$	absorption and transport extinction coefficients
τ, τ_0	current and total optical thickness of the particle
φ, ϕ	azimuth angles
ξ	asymmetry parameter of illumination
ψ_k, ζ_k	Riccati–Bessel functions
ς	parameters defined by Eq. (29)
ω	angular parameter defined by Eq. (16)
$\vec{\Omega}$	unit vector of direction

Subscripts and superscripts

a	absorption
h	hemispherical
k	order of mathematical functions
r	radial component
s	scattering
sph	spherically symmetric
tr	transport
λ	spectral dependent
θ, φ	angular components
e	external

The angular dependence of the radiation illuminating single particle is known from the solution of the radiation transfer problem for the disperse system as a whole.

Unfortunately, the general Mie solution for radiation field in a spherical particle is very time-consuming [16–18]. It is known that integral characteristics of radiation absorption by large particles can be calculated in the geometrical optics approximation [16,19–21]. This approximation is inapplicable for local values near the caustics [16,22,23]. The latter limitation is important for laser illumination [18,24,25] but not for diffuse thermal radiation considered in this paper.

The objective of the paper is to suggest an approximate analytical model of radiation absorption in large semi-transparent particles in the range of applicability of the geometrical optics approximation. The model should be applicable in a wide spectral range and for arbitrary illumination of a polydisperse system.

2. Approximate description of asymmetric illumination of single particle

The radiation transfer is usually calculated for a number of spectral intervals and the following values are obtained at each point of the computational region: the spectral radiation intensity $I_\lambda(\vec{r}, \vec{\Omega})$ and the spectral radiation flux $\vec{q}_\lambda(\vec{r}) = \int_{(4\pi)} I_\lambda(\vec{r}, \vec{\Omega})\vec{\Omega}\,\mathrm{d}\vec{\Omega}$ [1–3]. The function $I_\lambda(\vec{r},\vec{\Omega})$ determines illumination of a particle placed at point $\vec{r}$. In the general case, the angular dependences of radiation intensity are very complex and different for various spectral intervals. To simplify the problem for single particle, one can describe the illumination of the particle as axisymmetric and uniform one in two hemispheres:

$$I_\lambda(\vec{r},\vec{\Omega}) = \begin{cases} I_\lambda^-(\vec{r}), & \vec{q}_\lambda\vec{\Omega} < 0 \\ I_\lambda^+(\vec{r}), & \vec{q}_\lambda\vec{\Omega} > 0 \end{cases} \tag{1}$$

Functions I_λ^- and I_λ^+ are determined by the following relation [3]:

$$I_\lambda^\pm = I_\lambda^0/(2\pi) \pm q_\lambda/\pi \tag{2}$$

where

$$I_\lambda^0(\vec{r}) = \int_{(4\pi)} I_\lambda(\vec{r},\vec{\Omega})\,\mathrm{d}\vec{\Omega} \tag{3}$$

The approximation suggested simplifies radically the calculations for single particle: one needs only to solve the problem for uniform illumination of the particle from a hemisphere. This solution is described by function $p_\lambda^{\mathrm{h}}(r,\mu)$ (absorbed power per unit spectral interval), where $0 \leqslant r \leqslant a$ is the radial coordinate measured from the centre of particle, $\mu = \cos\theta$ (θ is measured from the spectral radiation flux direction). The corresponding function for illumination from two hemispheres is as follows:

$$P_\lambda(r,\mu) = p_\lambda^{\mathrm{h}}(r,\mu)I_\lambda^+ + p_\lambda^{\mathrm{h}}(r,-\mu)I_\lambda^- \tag{4}$$

The last equation can be rewrite in the form:

$$P_\lambda(r,\mu) = \frac{\xi_\lambda p_\lambda^{\mathrm{sph}}(r) + (1-\xi_\lambda)p_\lambda^{\mathrm{h}}(r,\mu)}{1+\xi_\lambda}\,\frac{I_\lambda^0}{\pi} \tag{5}$$

where $\xi_\lambda = I_\lambda^-/I_\lambda^+$ is the asymmetry parameter of illumination ($0 \leqslant \xi_\lambda \leqslant 1$), $p_\lambda^{\mathrm{sph}}(r) = p_\lambda^{\mathrm{h}}(r,\mu) + p_\lambda^{\mathrm{h}}(r,-\mu)$ is the power absorption function for spherically symmetric illumination of the particle.

In the general case, the direction of spectral radiation flux is different for various spectral intervals and one can write subscript λ at angular parameter μ: $P_\lambda(r,\mu_\lambda)$. As a result, the integral power distribution will be three-dimensional:

$$P(r,\vartheta,\varphi) = \int_0^\infty P_\lambda(r,\mu_\lambda)\,\mathrm{d}\lambda \tag{6}$$

The absorption distribution in a particle may be axisymmetric only in the case of not too complex picture of the radiation transfer in disperse system.

3. Solution based on the Mie theory

The Mie theory of interaction between a plane linearly polarized electromagnetic wave with a homogeneous spherical particle gives the following relations for the complex amplitudes of the wave electric field inside the particle [16,17]:

$$\begin{aligned}
E_r &= \frac{E_0\cos\varphi}{z^2}\sum_{k=1}^{\infty} \mathrm{i}^k(2k+1)d_k\psi_k(z)P_k^{(1)}(\mu)\\
E_\theta &= \frac{E_0\cos\varphi}{z}\sum_{k=1}^{\infty}\frac{\mathrm{i}^k(2k+1)}{k(k+1)}[c_k\psi_k(z)\pi_k(\mu) - \mathrm{i}d_k\psi_k'(z)\tau_k(\mu)]\\
E_\varphi &= \frac{E_0\sin\varphi}{z}\sum_{k=1}^{\infty}\frac{\mathrm{i}^{k+1}(2k+1)}{k(k+1)}[\mathrm{i}c_k\psi_k(z)\tau_k(\mu) + d_k\psi\prime_k(z)\pi_k(\mu)]\\
c_k &= \frac{m\mathrm{i}}{m\zeta_k(x)\psi_k'(y) - \zeta_k'(x)\psi_k(y)} \qquad d_k = \frac{m\mathrm{i}}{\zeta_k(x)\psi_k'(y) - m\zeta_k'(x)\psi_k(y)}\\
\pi_k &= \frac{P_k^{(1)}(\mu)}{\sqrt{1-\mu^2}} \qquad \tau_k = -\sqrt{1-\mu^2}\frac{\mathrm{d}}{\mathrm{d}\mu}P_k^{(1)}(\mu)
\end{aligned} \tag{7}$$

where E_0 is the amplitude of the electric field in the incident wave, $x = 2\pi a/\lambda$ is the diffraction parameter, $m = n - \mathrm{i}\kappa$ is the complex index of refraction, $y = mx$, $z = mx\bar{r}$, $\bar{r} = r/a$, $\mu = \cos\theta$, θ is the angle measured from the incident radiation direction, φ is the azimuth angle measured from the plane of electric field vibration of the incident wave, ψ_k, ζ_k are the Riccati–Bessel functions, $P_k^{(1)}$ are the associated Legendre polynomials. The dimensionless power absorption function can be determined by the following equation [18,19,26]:

$$\bar{p}_\lambda(r,\mu,\varphi) = \frac{4\pi n\kappa}{\lambda}\,\frac{|E_r|^2 + |E_\theta|^2 + |E_\varphi|^2}{|E_0|^2} \tag{8}$$

In the case of unpolarized external radiation, the integration of Eq. (8) over φ gives:

$$\bar{p}_\lambda(r,\mu) = \frac{8\pi^2 n\kappa}{\lambda}s(r,\mu) \tag{9}$$

where

$$\begin{aligned}
s(r,\mu) &= |e_r|^2 + |e_\theta|^2 + |e_\varphi|^2\\
|e_r|^2 &= \frac{1}{2}\left|\frac{1}{z^2}\sum_{k=1}^{\infty}\mathrm{i}^k(2k+1)d_k\psi_k(z)P_k^{(1)}(\mu)\right|^2\\
|e_\theta|^2 &= \frac{1}{2}\left|\frac{1}{z}\sum_{k=1}^{\infty}\frac{\mathrm{i}^k(2k+1)}{k(k+1)}[c_k\psi_k(z)\pi_k(\mu) - \mathrm{i}d_k\psi_k'(z)\tau_k(\mu)]\right|^2\\
|e_\varphi|^2 &= \frac{1}{2}\left|\frac{1}{z}\sum_{k=1}^{\infty}\frac{\mathrm{i}^k(2k+1)}{k(k+1)}[\mathrm{i}c_k\psi_k(z)\tau_k(\mu) + d_k\psi_k'(z)\pi_k(\mu)]\right|^2
\end{aligned} \tag{10}$$

If the illumination of the particle is spherically symmetric then the radial profile of the absorbed power can be found from the equation:

$$\begin{aligned}
\bar{p}_\lambda^{\mathrm{sph}}(r) &= \int_{-1}^{1}\bar{p}_\lambda(r,\mu)\,\mathrm{d}\mu = \frac{16\pi^2 n\kappa}{\lambda}S(r)\\
S(r) &= \frac{1}{2}\int_{-1}^{1}s(r,\mu)\,\mathrm{d}\mu
\end{aligned} \tag{11}$$

Analytical expression for function S can be found elsewhere [18]:

$$S = \frac{1}{2|z|^4}\sum_{k=1}^{\infty}(2k+1)\Big[k(k+1)|d_k\psi_k(z)|^2 + |z|^2\Big(|c_k\psi_k(z)|^2 + |d_k\psi_k'(z)|^2\Big)\Big] \tag{12}$$

Note that absorption efficiency factor Q_a can be determined as [20]:

$$Q_a(x,m) = 8\pi\kappa x\int_0^1 S(x,m,\bar{r})\bar{r}^2\,\mathrm{d}\bar{r} \tag{13}$$

In the case of uniform illumination of the particle from a hemisphere, the absorbed power per unit volume can be found from the relation [27]:

$$\bar{p}_\lambda^{\mathrm{h}}(r,\mu) = \frac{16\pi^2 n\kappa}{\lambda}S_{\mathrm{h}}(r,\mu) \tag{14}$$

where

$$S_{\mathrm{h}}(r,\mu) = \frac{1}{2\pi}\int_0^1\left[\int_0^{2\pi}s(r,\mu_0)\,\mathrm{d}\phi'\right]\mathrm{d}\mu' \qquad \mu_0 = \mu\mu' + \sqrt{1-\mu^2}\sqrt{1-\mu'^2}\cos(\phi-\phi') \tag{15}$$

Since $S_{\mathrm{h}}(r,\mu)$ does not depend on azimuth angle ϕ one can rewrite Eq. (15) as:

$$S_{\mathrm{h}}(r,\mu) = \frac{1}{\pi}\int_0^1\left[\int_{-1}^1\frac{s(r,\mu_0)}{\sqrt{1-\omega^2}}\,\mathrm{d}\omega\right]\mathrm{d}\mu' \qquad \mu_0 = \mu\mu' + \sqrt{1-\mu^2}\sqrt{1-\mu'^2}\,\omega \quad \omega = \cos\phi' \tag{16}$$

Note that $S_{\mathrm{h}}(r,\mu) + S_{\mathrm{h}}(r,-\mu) = S(r)$ and $S_{\mathrm{h}}(r,0) = S(r)/2$.

The radiation power absorbed by a droplet as a whole is characterized by the efficiency factor of absorption Q_a, which can be calculated independently by use of the known equations of the Mie theory. For large semi-transparent particles, one can use also approximate relation suggested in [28]. For this reason, it is sufficient to consider the normalized distribution of the absorbed power. In the case of spherically symmetric illumination of particles, this distribution is characterized by the following function:

$$w(\bar{r}) = \frac{S(\bar{r})}{3\int_0^1 S(\bar{r})\bar{r}^2\,\mathrm{d}\bar{r}} \tag{17}$$

In the case when the particle is illuminated from a hemisphere, the normalized distribution of the absorbed power depends on two variables:

$$w_{\mathrm{h}}(\bar{r},\mu) = \frac{S_{\mathrm{h}}(\bar{r},\mu)}{3\int_0^1\int_{-1}^1 S_{\mathrm{h}}(\bar{r},\mu)\bar{r}^2\,\mathrm{d}\mu\,\mathrm{d}\bar{r}} \tag{18}$$

It was shown in [20,21] that function $w(\bar{r})$ for large semi-transparent particles can be calculated in the geometrical optics approximation: solution of the radiation transfer equation inside the particle at $x \geqslant 20$ gives almost the same profiles of absorbed power as those calculated by using the Mie theory.

4. Modified differential approximation for symmetrically illuminated particles

In the case of spherically symmetric illumination of a particle, the radial distribution of absorbed power coincides with the profile of radiation power generated by isothermal particle. An analysis of the radiation transfer inside a homogeneous semi-transparent particle [20,21] showed different behaviour of the angular dependences of the radiation intensity. In the central zone $\bar{r} < \bar{r}_* = a/n$ this angular dependence can be well described by the usual DP_0-approximation of the double spherical harmonics [3], whereas in the periphery of the particle $\bar{r}_* < \bar{r} < 1$ the reflection of radiation from the particle surface leads to more complex angular pictures. To solve the problem in the whole volume of the particle, the author suggested a modified approximation called MDP_0. The derivation of MDP_0 equations for general case of nonisothermal particle and evaluation of the accuracy of this approximation can be found in [29]. It was shown that MDP_0 gives sufficiently accurate results for the radiation field inside a semi-transparent particle, and the computational time is two orders of magnitude less than that of the numerical solution of the radiation transfer equation. The latter circumstance is very important advantage in comparison with the ray tracing procedure [11,30] especially for numerical analysis of combined heat transfer problems.

In the case of illumination of a particle by external radiation, the boundary-value problem in MDP_0-approximation for function $g_0(\tau)$ can be written in the following form:

$$\frac{1}{\tau^2}(\tau^2 D_\lambda g_0')' - (1-\mu_*)g_0 = 0 \tag{19}$$

$$g_0'(0) = 0 \quad D_\lambda g_0'(\tau_0) = \frac{4n^2 - g_0(\tau_0)}{n(n^2+1)} \tag{20}$$

where

$$D_\lambda = \frac{1+\mu_*}{4}(1-\mu_*^2) \qquad \mu_* = \sqrt{1-(\tau_*/\tau)^2}\,\Theta(\tau-\tau_*) \quad \tau = \bar{r}\tau_0 \quad \tau_* = \bar{r}_*\tau_0 \tag{21}$$

$\tau_0 = 2\kappa x$ is the spectral optical thickness of the particle, Θ is the Heaviside unit step function. The dimensionless absorbed radiation power per unit spectral interval is determined as:

$$p_\lambda^{\mathrm{s}}(\tau) = (1-\mu_*)g_0(\tau) \tag{22}$$

The problem (19)–(22) for single particle can be easily solved numerically [5]. At the same time, a further simplification of the problem is important for possible implementation of the solution in multidimensional CFD codes and engineering calculations by taking into account combined heat and mass transfer processes. The simplest analytical approximation has been suggested in [31] separately for small and large optical thickness τ_0. This approximation is not good for intermediate values of the optical thickness, but it appears to be rather good for droplets of diesel fuel, which has wide regions of semi-transparency and sharp peaks of absorption. In the present paper, a more reliable approximate analytical solution for the whole range of the particle optical thickness is derived. To obtain this solution one can use the following approximate expression for radiation diffusion coefficient at the periphery of the particle ($\tau > \tau_*$):

$$D_\lambda = D_\lambda^{(1)} = \frac{1}{2}\left(\frac{\tau_*}{\tau}\right)^2 \tag{23}$$

Strongly speaking, this relation is correct only near the particle surface for $n^2 \gg 1$. Similarly, one can replace also coefficient $(1-\mu_*)$ in the second term of Eq. (19) by $D_\lambda^{(1)}$. As a result, Eq. (19) at $\tau > \tau_*$ is simplified radically:

$$g_0'' - g_0 = 0 \tag{24}$$

Having in mind the continuity of function g_0 and their derivative g_0' at $\tau = \tau_*$ one can find the following analytical solution:

$$\begin{aligned}
&\tau \leqslant \tau_* \quad p_\lambda^{\mathrm{sph}} = g_0 = A\frac{\tau_*}{\tau}\frac{\sinh(2\tau)}{\sinh(2\tau_*)}(\sinh\tau_* + \zeta\cosh\tau_*)\\
&\tau > \tau_* \quad p_\lambda^{\mathrm{sph}} = \left(1-\sqrt{1-(\tau_*/\tau)^2}\right)g_0\\
&\quad g_0 = A(\sinh\tau + \zeta\cosh\tau)
\end{aligned} \tag{25}$$

where

$$\begin{aligned}
&A = \frac{4n^2 v}{v(\sinh\tau_0 + \zeta\cosh\tau_0) + (\cosh\tau_0 + \zeta\sinh\tau_0)}\\
&\zeta = \frac{\gamma\tanh\tau_* - 1}{\tanh\tau_* - \gamma} \quad \gamma = \frac{2}{\tanh\tau_*} - \frac{1}{\tau_*} \quad v = \frac{2n}{n^2+1}
\end{aligned} \tag{26}$$

For large optical thickness, Eqs. (25) and (26) can be considerably simplified:

$$\begin{aligned}
&\tau \leqslant \tau_* \quad p_\lambda^{\mathrm{sph}} = \frac{2n^2 v}{1+v}\left(\frac{\tau_*}{\tau}\right)\exp(2\tau - \tau_* - \tau_0)\\
&\tau > \tau_* \quad p_\lambda^{\mathrm{sph}} = \frac{2n^2 v}{1+v}\left(1-\sqrt{1-(\tau_*/\tau)^2}\right)\exp(\tau - \tau_0)
\end{aligned} \tag{27}$$

The last equations are applicable for calculations at $\tau_0 > 5$. The normalized spectral profiles of absorption are calculated as:

$$w(\bar r) = \frac{p_\lambda^{\mathrm{sph}}(\bar r)}{3\int_0^1 p_\lambda^{\mathrm{sph}}(\bar r)\bar r^2\,\mathrm{d}\bar r} \tag{28}$$

5. Evaluation of the accuracy of approximate solution for symmetric illumination

The calculated radial profiles of the absorbed radiation power $w(\bar r)$ are presented in Fig. 1. Having in mind possible applications of this research to droplets of water or typical fuels, the values of refraction index $n = 1.3$ and $n = 1.5$ are considered. The Mie theory calculations at $x = 50$ are also shown in Fig. 1. Remember that Mie solution at $x > 20$ is close to the geometrical optics limit [20,21]. Note that the kink in the curves at $\bar r = \bar r_*$ can be easily explained in terms of the geometrical optics [20,32].

One can see that approximate analytical solution (25) and (26) gives absorption profiles, which practically

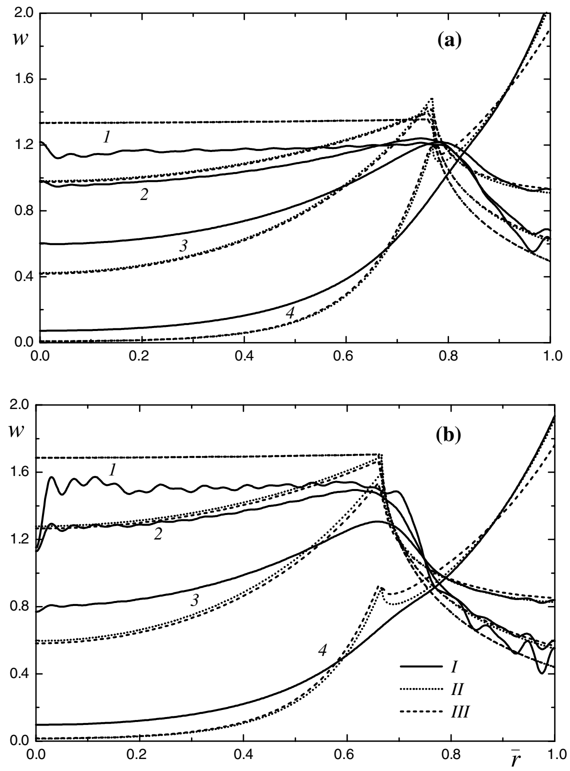

Fig. 1. Normalized radial profiles of the absorbed radiation power inside droplets in the case of their symmetric illumination: (a) $n = 1.3$, (b) $n = 1.5$; I—Mie calculations for $x = 50$, II—numerical solution in MDP_0-approximation, III—approximate analytical solution; 1—$\tau_0 = 0.2$, 2—$\tau_0 = 1$, 3—$\tau_0 = 2$, 4—$\tau_0 = 5$.

coincide with those obtained by numerical solution of the boundary-value problem (19)–(22). In contrast to approximate relations suggested in [31], the solution (25) and (26) is applicable at arbitrary optical thickness of the particle.

6. Approximation of Mie calculations for illumination from a hemisphere

By analysis of calculations for illumination from a hemisphere, it is convenient to use the relation $S_h(\bar{r},0) = S(\bar{r})/2$ and consider only the function $\bar{S}_h(\bar{r},\mu) = S_h(\bar{r},\mu)/S_h(\bar{r},0)$. The results of Mie calculations for $x = 50$ at several fixed values of $\bar{r}$ are presented in Figs. 2 and 3. In the case of small optical thickness (Fig. 2), the radiation is absorbed mainly in the shadowed part of the particle. The strongest angular dependence of absorption is predicted near the particle surface ($\bar{r} > \bar{r}_*$) in the vicinity of the plane $\mu = 0$. The increase of the optical thickness leads to an increase in radiation absorption in the illuminated part of the particle and the dependence of the radiation absorption on μ becomes monotonic. In the case of large optical thickness (Fig. 3), the radiation is absorbed mainly near the illumi-

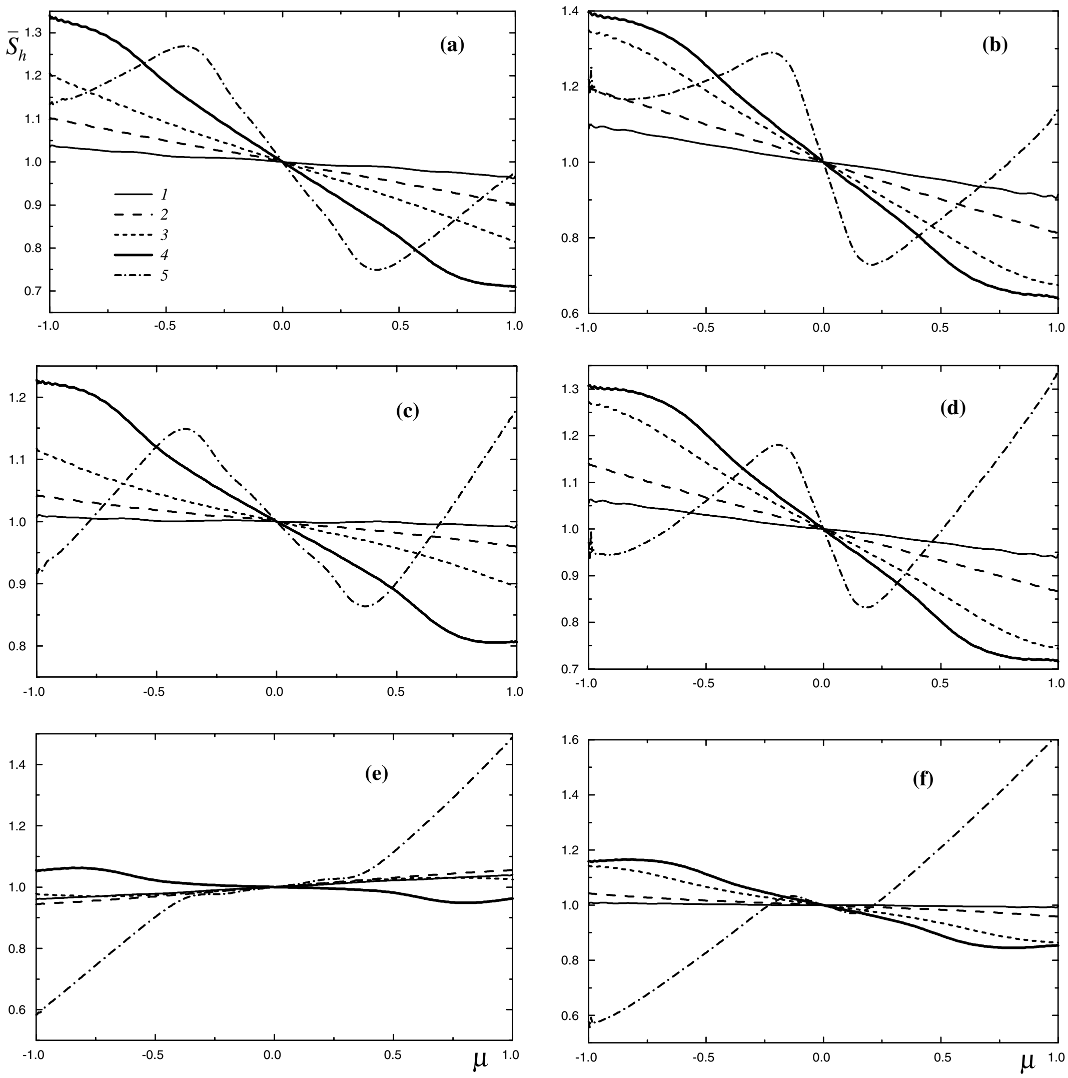

Fig. 2. Angular distribution of the absorbed radiation power inside optically thin droplet in the case of its illumination from a hemisphere for $n = 1.3$ (a,c,e) and $n = 1.5$ (b,d,f): (a,b) $\tau_0 = 0.2$; (c,d) $\tau_0 = 0.5$; (e,f) $\tau_0 = 1$; 1—$\bar{r} = 0.2$, 2—$\bar{r} = 0.4$, 3—$\bar{r} = 0.6$, 4—$\bar{r} = 1/n$, 5—$\bar{r} = 1$.

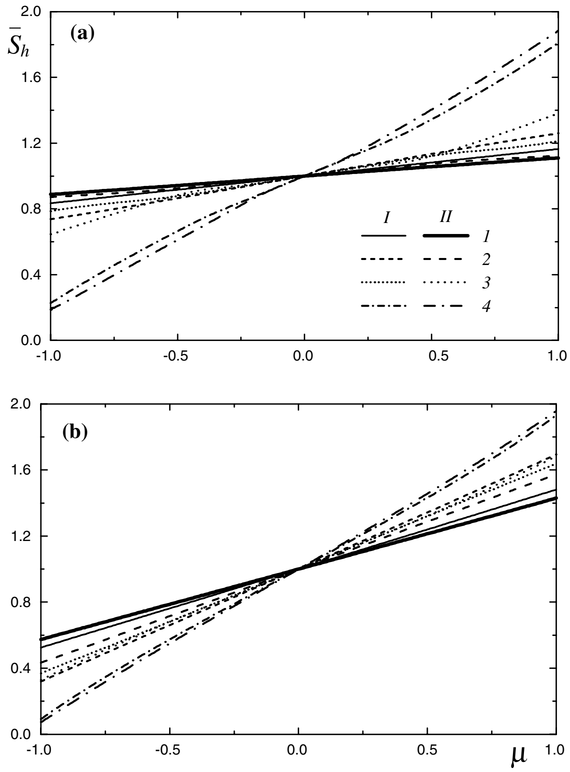

Fig. 3. Angular distribution of the absorbed radiation power inside optically thick droplet in the case of its illumination from a hemisphere for $n = 1.3$ (plots I) and $n = 1.5$ (plots II): (a) $\tau_0 = 2$, (b) $\tau_0 = 5$; 1—$\bar{r} = 0.25$, 2—$\bar{r} = 0.5$, 3—$\bar{r} = 0.75$, 4—$\bar{r} = 1$.

nated surface of the particle and the dependence on μ becomes almost linear.

Following [33], note that $\bar{S}_h(\bar{r}, \mu) - 1$ can be approximated by an odd function of μ (see Figs. 2 and 3). This approximation simplifies considerably the integration of function $S_h(\bar{r}, \mu)$:

$$\int_{-1}^{1} S_h(\bar{r}, \mu)\,d\mu = S(\bar{r}) \tag{29}$$

As a result, one can write:

$$w_h(\bar{r}, \mu) = w(\bar{r})\bar{S}_h(\bar{r}, \mu)/2 \tag{30}$$

The analytical approximation of function $\bar{S}_h(\bar{r}, \mu)$ can be found in the form

$$\bar{S}_h(\bar{r}, \mu) = 1 + \mu f(n, \bar{r}, |\mu|) \tag{31}$$

At optical thickness of the particle $\tau_0 \geqslant 2$, the function f can be treated as independent on index of refraction n and angular coordinate μ (see Fig. 3):

$$f(\bar{r}) = 1 - (1 - \bar{r}^{2+2/\tau_0}) \exp[(2 - \tau_0)/6] \tag{32}$$

Eq. (32) gives a correct result in the limit of great optical thickness. At small optical thickness ($\tau_0 < 2$) the following approximation is suggested:

$$\begin{aligned} f(n, \bar{r}, |\mu|) &= (n-1)[B(\bar{r}) - C(\bar{r})(1 - |\mu|)\Theta(\bar{r} - \bar{r}_*)] \\ B(\bar{r}) &= B_1 + (B_2 - B_1)\left[1 - \left(\frac{1-\bar{r}}{1-\bar{r}_*}\right)^2\right]\Theta(\bar{r} - \bar{r}_*) \\ C(\bar{r}) &= \frac{1.5}{n-1}\left(\frac{\bar{r}-\bar{r}_*}{1-\bar{r}_*}\right)\frac{1}{1+\tau_0^2} \\ B_1 &= -\frac{\bar{r}}{1+\tau_0^2} \quad B_2 = \frac{1 - \exp[-1.5(n-1)\tau_0]}{n-1} \end{aligned} \tag{33}$$

Approximate relations (31)–(33) are more correct than those suggested in [33] and can be used at arbitrary optical thickness of the particle. The results of approximate calculations at the same parameters as in Figs. 2 and 3 are presented in Figs. 4 and 5. One can see that Eqs. (31)–(33) give a reasonable approximation of the exact calculations.

7. Calculations for water and diesel fuel droplets

Consider a one-dimensional model problem for optically thick homogeneous disperse system in the region with flat boundary surface illuminated by diffuse thermal radiation. In this case, one can use the simplest DP_0-approximation for the radiation transfer calculation. The corresponding boundary-value problem for function $g_0 = I_\lambda^-(y) + I_\lambda^+(y)$ (here y is the coordinate measured from the illuminated surface of the disperse system) is as follows [3,34]:

$$\begin{aligned} &\frac{1}{4\Sigma_{tr}} g_0'' - \Sigma_a g_0 = 0 \\ &g_0'(0) = 2\Sigma_{tr}[g_0(0) - 4q_\lambda^e] \quad g_0(\infty) = 0 \end{aligned} \tag{34}$$

where q_λ^e is the spectral external radiation flux, Σ_a, Σ_{tr} are the absorption and transport extinction coefficients of the medium. The spectral radiation flux is determined as:

$$q_\lambda = (I_\lambda^+ - I_\lambda^-)/2 = -g_0'/(4\Sigma_{tr}) \tag{35}$$

The analytical solution of the problem (34) and (35) is as follows:

$$\begin{aligned} g_0(y) &= \frac{4q_\lambda^e}{1+\sqrt{\alpha_\lambda}} \exp(-2\sqrt{\alpha_\lambda}\Sigma_{tr} y) \\ q_\lambda(y) &= \frac{\sqrt{\alpha_\lambda}}{2} g_0(y) \quad \alpha_\lambda = \Sigma_a/\Sigma_{tr} \end{aligned} \tag{36}$$

and one can obtain very simple equation for asymmetry parameter of illumination:

$$\xi_\lambda = \frac{1 - \sqrt{\alpha_\lambda}}{1 + \sqrt{\alpha_\lambda}} \tag{37}$$

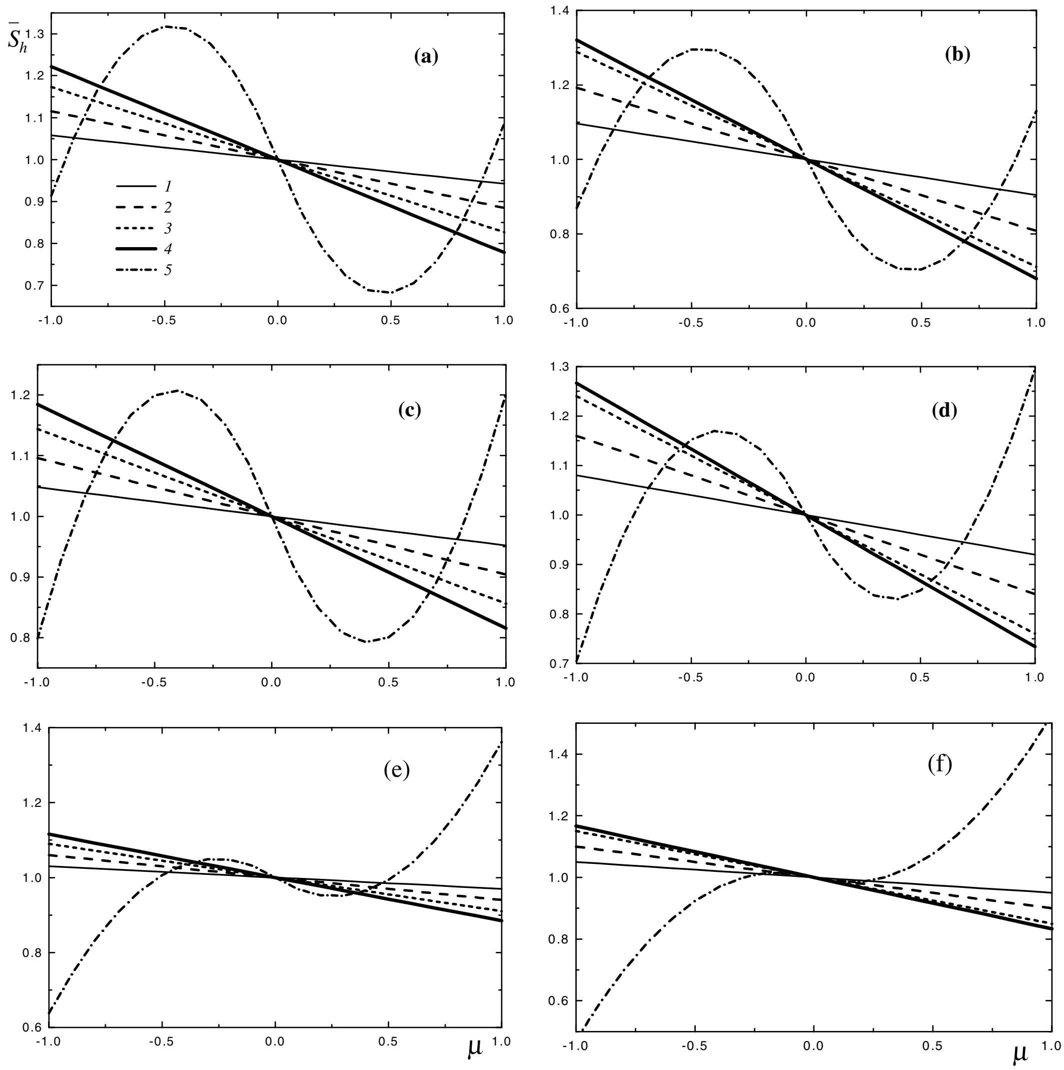

Fig. 4. Approximate angular distribution of the absorbed radiation power inside optically thin droplet in the case of its illumination from a hemisphere. Calculations by use of Eqs. (31)–(33). Designations see in Fig. 2.

Note that ξ_λ do not depend on coordinate and coincides with the reflection coefficient of the disperse system.

For polydisperse systems of large droplets of water or diesel fuel, the value of α_λ can be calculated in monodisperse approximation using the droplet radius $a = a_{32} = \int_0^\infty a^3 F(a)\,\mathrm{d}a \big/ \int_0^\infty a^2 F(a)\,\mathrm{d}a$, where $F(a)$ is the size distribution function [3,35]:

$$\alpha_\lambda = Q_\mathrm{a}/Q_\mathrm{tr} \tag{38}$$

where Q_s^tr, $Q_\mathrm{tr} = Q_\mathrm{a} + Q_\mathrm{s}^\mathrm{tr}$ are the transport efficiency factors of scattering and extinction. The following approximate relations can be used for both water and fuel droplets [28,35]:

$$Q_\mathrm{a} = \frac{4n}{(n+1)^2}[1 - \exp(-2\tau_0)]$$

$$Q_\mathrm{s}^\mathrm{tr} = \begin{cases} C\varsigma, & \varsigma \leqslant 1 \\ C/\varsigma^\gamma, & \varsigma > 1 \end{cases} \quad C = 1.5n(n-1)\exp(-15\kappa)$$

$$\gamma = 1.4 - \exp(-80\kappa) \quad \varsigma = 0.4(n-1)x \tag{39}$$

To calculate ξ_λ by use of Eqs. (37)–(39) one needs the spectral data on optical constants $n(\lambda)$ and $\kappa(\lambda)$. The tabulated data of [36] for water and approximate relations of [31] for a typical diesel fuel are used in this paper. The corresponding spectral dependences of optical constants in the near infrared are shown in Fig. 6. The calculated functions $\xi_\lambda(\tau_0)$ are presented in Fig. 7. The step

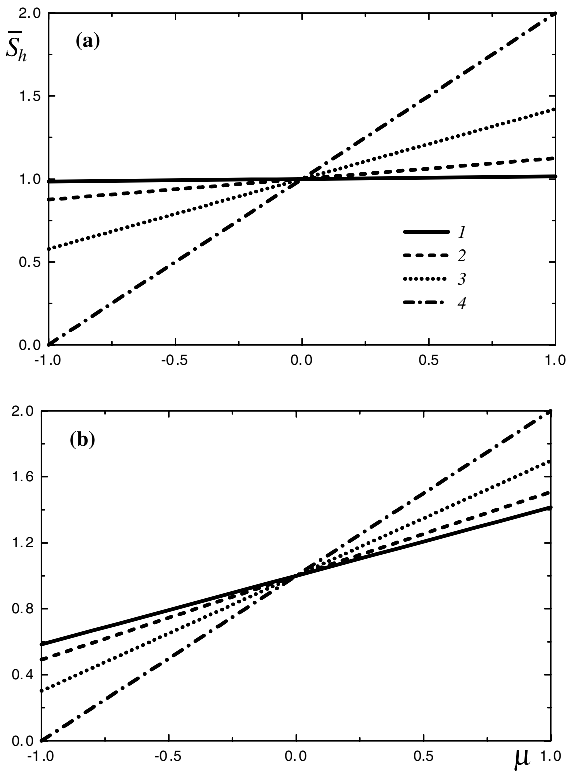

Fig. 5. Approximate angular distribution of the absorbed thermal radiation power inside optically thick droplet in the case of its illumination from a hemisphere. Calculations by use of Eqs. (31)–(33): (a) $\tau_0 = 2$, (b) $\tau_0 = 5$; 1—$\bar{r} = 0.25$, 2—$\bar{r} = 0.5$, 3—$\bar{r} = 0.75$, 4—$\bar{r} = 1$.

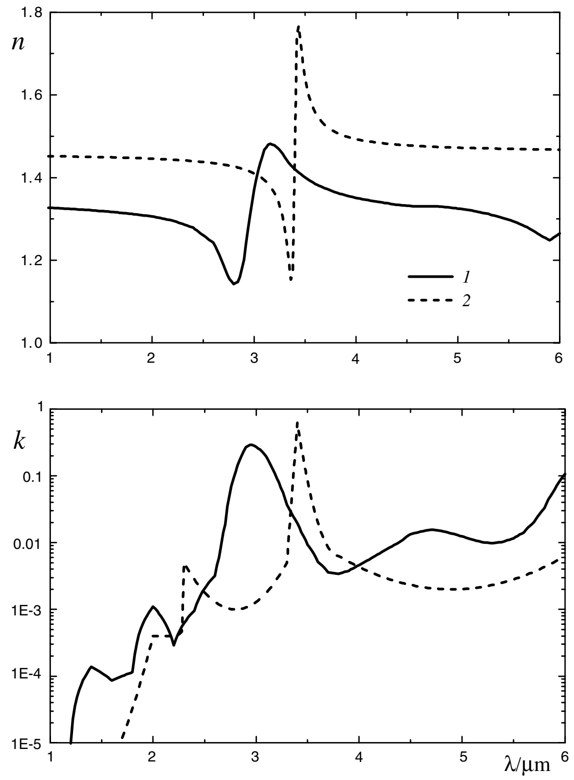

Fig. 6. Spectral optical constants of water (plots 1) and diesel fuel (plots 2) used in calculations.

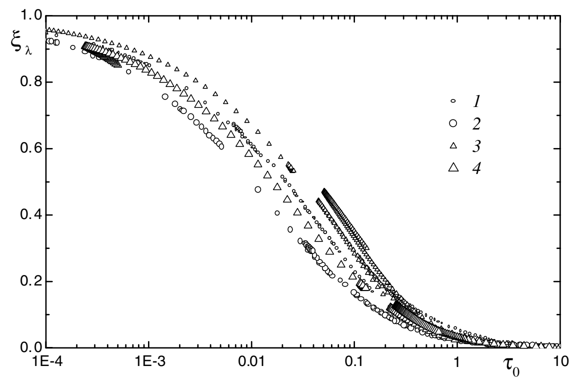

Fig. 7. Asymmetry parameter of droplet illumination in monodisperse optically thick medium. Calculations for water (1,2) and diesel fuel droplets (3,4) by use of Eqs. (37)–(39): (1,3) $a = 10\,\mu$m; (2,4) $a = 50\,\mu$m.

$\Delta\lambda = 0.02\,\mu$m by wavelength was used in the range $0.5 \leqslant \lambda \leqslant 6\,\mu$m. The values $\xi_\lambda > 0.1$ correspond to the short-wave range ($\lambda < 2.3\,\mu$m for water and $\lambda < 3\,\mu$m for fuel), where the droplets are almost transparent. For various droplets, the function $\xi_\lambda(\tau_0)$ can be approximated as:

$$\xi_\lambda = \exp(-\beta\tau_0) \tag{40}$$

where $\beta = 4$ for small fuel droplets, $\beta = 5$ for large fuel droplets and small water droplets, and $\beta = 6$ for large water droplets. Note that approximate relations for optical constants of diesel fuel [31] give the minimal evaluation for $\kappa(\lambda)$ in the important spectral range $1.1 < \lambda < 2\,\mu$m [35]. Additional measurements might show the greater values of the index of absorption in this spectral range. In this case, the values of parameter β for fuel droplets could be the same as for water droplets. Remember that one should use the value of τ_0 for equivalent radius of droplets a_{32} in Eq. (40), and the value of ξ_λ is the same for droplets of different radius.

In the case of a blackbody spectrum of external thermal radiation, the normalized radiation power absorbed in a particle is calculated as follows:

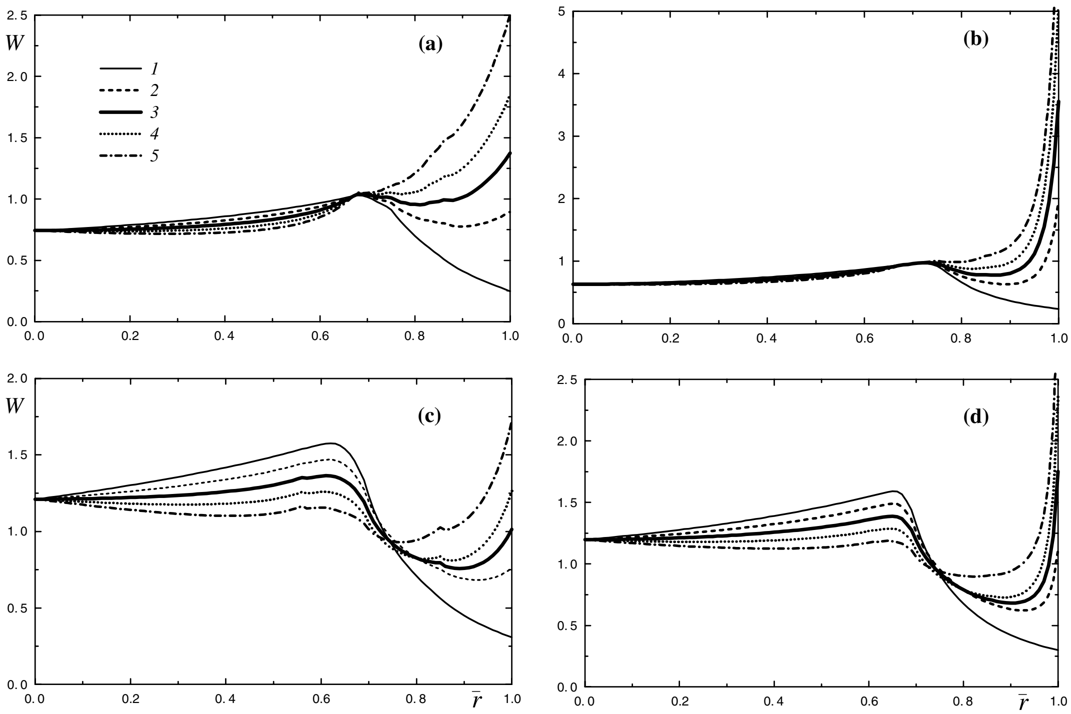

Fig. 8. Normalized radial profiles of the absorbed thermal radiation power inside droplets of water (a, b) and diesel fuel (c, d) in the case of optically thick disperse system: (a, c) $a = 10\,\mu\text{m}$; (b, d) $a = 50\,\mu\text{m}$; 1—$\mu = -1$ (the side facing the disperse system), 2—$\mu = -0.5$, 3—$\mu = 0$, 4—$\mu = 0.5$, 5—$\mu = 1$ (the side facing the external radiation).

$$W(\bar{r}, \mu) = \int_{\lambda_1}^{\lambda_2} [\xi_\lambda w(\bar{r}) + (1 - \xi_\lambda) w_h(\bar{r}, \mu)] \times Q_a B_\lambda(T_e)\, d\lambda \bigg/ \int_{\lambda_1}^{\lambda_2} Q_a B_\lambda(T_e)\, d\lambda \tag{41}$$

The results of calculations for water and fuel droplets at $a_{32} = 50\,\mu\text{m}$ and $T_e = 1500\,\text{K}$ are presented in Fig. 8. The plots for $\mu = 0$ coincide with the absorption profiles for spherically symmetric illumination. One can see that thermal radiation is absorbed mainly in the central zone and in the thin layer close to the droplet surface. Increased absorption in the central zone is related to the contribution of radiation in the semi-transparency ranges, where the droplet thickness is small. Significant absorption near the droplet surface facing the external radiation is related to the contribution of radiation near the absorption peaks of the droplet substance. The main peak of absorption for diesel fuel is placed at the wavelength $\lambda = 3.4\,\mu\text{m}$ that is in the more long-wave region in comparison with the absorption peak for water (see Fig. 6). It is far from the maximum of the external radiation ($\lambda = 2\,\mu\text{m}$). As a result, the absorption of radiation near the droplet surface facing the external radiation is not as strong for fuel droplets as that for droplets of water. At the same time, the absorption in the central zone of diesel fuel droplets is not symmetric: it is greater at the "shadow" side facing the droplet system.

8. Conclusions

An approximate theoretical model of radiation absorption in comparably cold semi-transparent spherical particles for the case of arbitrary illumination of a disperse system by external thermal radiation is suggested. After the usual spectral calculation of the radiation transfer in the disperse system, the asymmetric illumination of single particles is considered approximately as a uniform illumination from two hemispheres oriented according the direction of spectral radiation flux. As a result, the general problem reduces to that for illumination of the particle from a hemisphere. The Mie theory and the modified differential approximation (for symmetrically illuminated large particles) are employed. Approximate analytical relations are obtained for distribution of absorbed radiation power in a large spherical particle of arbitrary optical thickness. These relations are applicable for a typical range of refractive index of particle substance.

An analytical solution of model problem for optically thick polydisperse system is obtained. The results of calculations for disperse systems of water and diesel fuel droplets are presented. Thermal radiation is absorbed mainly in the central zone of droplets and near the droplet surface facing the external radiation. The first effect is related to contribution of semi-transparency ranges, whereas the surface absorption corresponds to absorption peaks of the droplet substance.

Acknowledgments

The work was partially supported by the Royal Society and by Russian Foundation for Basic Research (grant no. 04-02-16014).

References

[1] R. Siegel, J.R. Howell, Thermal Radiation Heat Transfer, Hemisphere, New York, 1992 (Chapter 21).
[2] R. Viskanta, M.P. Mengüç, Radiative transfer in dispersed media, Appl. Mech. Rev. 42 (9) (1989) 241–259.
[3] L.A. Dombrovsky, Radiation Heat Transfer in Disperse Systems, Begell House, New York, 1996 (Chapters 1 and 2).
[4] J.K. Fiszdon, Melting of powder grains in a plasma flame, Int. J. Heat Mass Transfer 22 (5) (1979) 749–761.
[5] L.A. Dombrovsky, M.B. Ignatiev, An estimate of the temperature of semitransparent oxide particles in thermal spraying, Heat Transfer Eng. 24 (2) (2003) 60–68.
[6] P.L.C. Lage, R.H. Rangel, Single droplet vaporization including thermal radiation absorption, J. Thermophys. Heat Transfer 7 (3) (1993) 502–509.
[7] P.L.C. Lage, C.M. Hackenberg, R.H. Rangel, Nonideal vaporization of dilating binary droplets with radiation absorption, Combust. Flame 101 (1) (1995) 36–44.
[8] S.K. Aggarwal, A review of spray ignition phenomena: present status and future research, Progr. Energy Combust. Sci. 24 (6) (1998) 565–600.
[9] L.A. Dombrovsky, L.I. Zaichik, Conditions of thermal explosion of a radiating gas with polydisperse liquid fuel, High Temp. 39 (4) (2001) 604–611.
[10] L.A. Dombrovsky, S.S. Sazhin, E.M. Sazhina, G. Feng, M.R. Heikal, M.E.A. Bardsley, S.V. Mikhalovsky, Heating and evaporation of semi-transparent diesel fuel droplets in the presence of thermal radiation, Fuel 80 (11) (2001) 1535–1544.
[11] G.M. Harpole, Radiative absorption by evaporating droplets, Int. J. Heat Mass Transfer 23 (1) (1980) 17–26.
[12] T.S. Ravigururajan, M.R. Beltran, A model for attenuation fire radiation through water droplets, Fire Safety J. 15 (2) (1989) 171–181.
[13] S. Dembele, A. Delmas, J.-F. Sacadura, A method of modeling the mitigation of hazardous fire thermal radiation by water spray curtains, ASME J. Heat Transfer 119 (4) (1997) 746–753.
[14] G. Grant, J. Brenton, D. Drysdale, Fire suppression by water sprays, Progr. Energy Combust. Sci. 26 (2) (2000) 79–130.
[15] G. Miliauskas, Regularities of unsteady radiative-conductive heat transfer in evaporating semitransparent liquid droplets, Int. J. Heat Mass Transfer 44 (4) (2001) 785–798.
[16] H.C. Van de Hulst, Light Scattering by Small Particles, Wiley, NY, 1957 (Chapters 9 and 11).
[17] C.F. Bohren, D.R. Huffman, Absorption and Scattering of Light by Small Particles, Wiley, New York, 1983 (Chapters 4 and 7).
[18] A.P. Prishivalko, Optical and Thermal Fields in Light Scattering Particles (in Russian), Nauka i Tekhnika, Minsk, 1983 (Chapter 1).
[19] A. Tuntomo, C.L. Tien, S.H. Park, Internal distribution of radiant absorption in a spherical particle, ASME J. Heat Transfer 113 (2) (1991) 407–412.
[20] L.A. Dombrovsky, Thermal radiation of a spherical particle of semitransparent material, High Temp. 37 (2) (1999) 260–269.
[21] L.A. Dombrovsky, Thermal radiation from nonisothermal spherical particles of a semitransparent material, Int. J. Heat Mass Transfer 43 (9) (2000) 1661–1672.
[22] J.A. Lock, E.A. Hovenac, Internal caustic structure of illuminated liquid droplets, J. Opt. Soc. Am. A 8 (10) (1991) 1541–1552.
[23] D.Q. Chowdhury, P.W. Barber, S.C. Hill, Energy–density distribution inside large nonabsorbing spheres by using Mie theory and geometrical optics, Appl. Opt. 31 (18) (1992) 3518–3523.
[24] L.G. Astafieva, A.P. Prishivalko, Heating of solid aerosol particles exposed to intense optical radiation, Int. J. Heat Mass Transfer 41 (2) (1998) 489–499.
[25] B.-S. Park, R.L. Armstrong, Laser droplet heating; fast and slow heating regimes, Appl. Opt. 28 (17) (1989) 3671–3680.
[26] D.W. Mackowski, R.A. Altenkirch, M.P. Mengüç, Internal absorption cross sections in a stratified sphere, Appl. Opt. 29 (10) (1990) 1551–1559.
[27] P.L.C. Lage, R.H. Rangel, Total thermal radiation absorption by a single spherical droplet, J. Thermophys. Heat Transfer 7 (1) (1993) 101–109.
[28] L.A. Dombrovsky, A spectral model of absorption and scattering of thermal radiation by droplets of diesel fuel, High Temp. 40 (2) (2002) 242–248.
[29] L.A. Dombrovsky, A modified differential approximation for thermal radiation of semitransparent nonisothermal particles: application to optical diagnostics of plasma spraying, J. Quant. Spectrosc. Radiat. Transfer 73 (2–5) (2002) 433–441.
[30] L.H. Liu, H.P. Tan, T.W. Tong, Internal distribution of radiation absorption in a semitransparent spherical particle, J. Quant. Spectrosc. Radiat. Transfer 72 (6) (2002) 747–756.
[31] L.A. Dombrovsky, S.S. Sazhin, Absorption of thermal radiation in a semi-transparent droplet: a simplified model, Int. J. Heat Fluid Flow 24 (6) (2003) 919–927.
[32] H.M. Lai, P.T. Leung, K.L. Poon, K. Young, Characterization of the internal energy density in Mie scattering, J. Opt. Soc. Am. A 8 (10) (1991) 1553–1558.

[33] L.A. Dombrovsky, S.S. Sazhin, Absorption of external thermal radiation in asymmetrically illuminated droplets, J. Quant. Spectrosc. Radiat. Transfer 87 (2) (2004) 119–135.

[34] L.A. Dombrovsky, Approximate methods for calculating radiation heat transfer in dispersed systems, Thermal Eng. 43 (3) (1996) 235–243.

[35] L.A. Dombrovsky, S.S. Sazhin, S.V. Mikhalovsky, R. Wood, M.R. Heikal, Spectral properties of diesel fuel droplets, Fuel 82 (1) (2003) 15–22.

[36] G.M. Hale, M.P. Querry, Optical constants of water in the 200 nm to 200 μm wavelength region, Appl. Opt. 12 (3) (1973) 555–563.